On the Ontology of Spacetime in a Frame of Reference

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Abstract

The spacetime ontology is considered in General Relativity (GR) in view of the choice of a frame of reference (FR). Various approaches to a description of the FR, such as coordinate systems, monads and tetrads are reviewed. It is shown that any of the existing FR definitions require a preexisting background spacetime, which, if defined independently of the FR, renders the spacetime absolute in violation of the principle of relativity, or, if defined within an inertial FR (IFR), as it is usually done, makes the argument circular. Consequently, defining a FR in a preexisting spacetime is unacceptable. We show that a FR defines a differentiable manifold with, generally, non-Euclidean geometry. In a noninertial FR (NIFR) the observer must chose a coordinative definition either admitting existence of a universal – inertial – force or settling for non-Euclidean spacetime. Following Reichenbach, it is preferable to eliminate all universal forces and opt for a non-Euclidean geometry. It is shown that a metric-affine space (L_4,g) is best suited to describe the geometry of spacetime within a FR. Considering a gravitational field in an arbitrary FR, we show within the framework of Einstein's GR that the gravity is described by nonmetricity of spacetime. This result may shed new light on the nature of the cosmological constant and dark energy.

I. Introduction

One of the fundamental problems of spacetime ontology is how matter affects the geometry of spacetime and, vice versa, how spacetime affects the behavior of the matter therein. Another problem is the emergence of spacetime in the frame of reference of an observer, i.e. how an observer affects (or, perhaps, creates) the spacetime, or its geometry. As we shall demonstrate here, these two problems are closely related.

II. Spacetime-Matter Interaction in General Relativity

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The classical General Relativity Theory (GRT) offers the following answers to the above stated questions:

1. The matter (and nongravitational fields) affect the metric ***g*** of the Riemannian space (V^4) by way of the Einstein field equation **G**=8π**T** or in coordinate representation:

$$G_{\mu\nu} \equiv R_{\mu\nu} + \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{1}$$

   where **G** (in a local chart x, $G_{\mu\nu}$) is the Einstein curvature tensor and **T** ($T_{\mu\nu}$) is the energy momentum tensor of matter and all nongravitational fields, $R_{\mu\nu}$ is the Ricci tensor, R is the scalar curvature and $g_{\mu\nu}$ is the metric tensor. We use here geometrical units, in which Newton's gravitational constant and the speed of light constant c are set to a unity.

2. The test particles move along the geodesic lines of Riemannian space, which are defined by the Levi-Civita (metric) connection Γ of V_4: $\nabla_x X = 0$, or in a local chart x

$$\frac{d^2 x^\lambda}{ds^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 0 \tag{2}$$

   where ∇ is a covariant derivative with respect to the Levi-Civita connection Γ ($\Gamma^\lambda_{\mu\nu}$) and τ is an affine parameter on the curve which we can take to mean proper time.

3. A frame of reference (FR) is represented by a coordinate system. An inertial frame of reference (IFR) is represented by a global Lorentz coordinate system while a noninertial frame of reference (NFR) is represented by a curvilinear coordinate system, which is locally Lorentz. Thus the principle of relativity, which initially was thought to be the cornerstone of GR, is reduced in GR to a trivial requirement of general covariance with respect to coordinate transformation.

In other words, Einstein's GR describes how matter curves spacetime by affecting its *metric*, which in turn, through a *metric-compatible* connection, tells matter how to move ([1], p. 5).

Clearly, GRT reduces frames of reference to coordinate systems, which play little role in the geometry of spacetime. This position is untenable because coordinate systems have no physical meaning whatsoever, while the frame of reference is a fundamental physical concept. A particular choice of a FR affects the physical laws therein.

As has been pointed out by Kretschman [2]; Fock [3], [4]; Wigner [7]; Rodichev [8], [9], [10], [11], [12]; Mitzkevich [22] and a few other authors, including this author [13], [14]; the coordinate system is merely a way to number points or label events of spacetime. Akin to street names and building numbers in a city, the coordinates are at best a convenience device ([1], p. 6-8). Any coordinate transformation affects the physics of spacetime no more than renaming a street or renumbering the houses on a street affects the lives of people who live therein. As Ohanian and Ruffini put it, "From a mathematical point of view, the covariance principle is therefore seen to be a triviality" ([15], p.373). We can well formulate both the geometry of spacetime and the physics in a given spacetime in the coordinate-free language of contemporary mathematics. (For example, the Einstein equations (1) as well as the equations for geodesic lines (2) above are given both coordinate and coordinate-free form.) Notwithstanding the obviousness of these arguments, the erroneous notion that coordinate systems describe reference frames stubbornly persists.

III. Spacetime in a Frame of Reference

A frame of reference, on the other hand, is one of the most important concepts of physics. Moreover, the epistemological importance of a reference frame cannot be overstated. One can say little about the state of a physical system in mechanics or field theory until one specifies the frame of reference in which said system is observed. Moreover, as Reichenbach pointed out [16], even the geometry of spacetime remains undetermined until we choose our coordinative definitions, such as the unit of length and the congruence of the standard units. It is the observer in a NIFR that can choose to entertain inertial ("universal", according to Reichenbach) forces or set them to zero, as recommended by Reichenbach, thereby forcing non-Euclidean geometry on spacetime [16].

Various observations conducted by different observers can only be compared if the reference frames of these observers are known along with laws of transformation, such as the Galilean transformation of the Newtonian mechanics, the Lorentz transformation for the IFR in special relativity, and yet to be determined transformations for the NIFR.

1. Problem of Measurement and Dimensionality of Space

A reference frame plays an important role in a measurement problem bearing upon the question of dimensionality of our space. The latter question cannot be resolved until it is clarified whether the space in question is a conceptual mathematical space or an empirical physical space. If Minkowski spacetime is viewed as a mathematical space, its 4-dimensionality presents no problem and merely signifies that we need four numbers (three spatial coordinates and a moment in time) to describe an event. As each physical event is characterized by some energy value, it is only logical to assign this value as the fifth coordinate thereby arriving at a five-dimensional space *a la* Kaluza-Klein.

Alternatively, the knowledge of the spatial coordinates of a test particle in a given moment is not enough to predict its motion, which requires also the knowledge of the velocity. Why not then add three more coordinates corresponding to the three components of the velocity vector to the description of each point, raising the dimensionality of spacetime to six! In fact, n-dimensional (where n is number of system parameters) configuration space is used in Lagrange formalism and $2n$-dimensional phase spaces are routinely used in Hamiltonian mechanics (let alone Hilbert space with its infinite number of dimensions used in quantum physics). Yet all of these constructs are well founded and legitimately used as a *conceptual* space.

To discuss the dimensionality of physical space, we must first define what we mean by physical space. In contrast to conceptual mathematical space, physical space is defined as *empirical* space whose geometry is determined by *measurement*. Consequently, the dimensionality of physical space must be demonstrable by our ability to directly measure such space. It is easy to see that we cannot directly measure therein Minkowski spacetime. We use rods to measure linear length (one-dimensional space); we can use standard squares or triangles to measure the area (two-dimensional space); we can also use standard cubes to measure the volume (three-dimensional space). But we cannot in principle construct a "standard event" to measure the volume of Minkowski spacetime. This seems to indicate that Minkowski space is a conceptual space and that the physical space is three-dimensional.

The problem that now arises is how to convert four-dimensional quantities of Special Relativity, which can never be observed or measured directly, into the observable three-dimensional objects. This can only be done in a given reference frame. Whatever the definition of the frame of reference, it must include a reference body wherein the measurement devices are situated. The worldline of this reference body is uniquely represented by its velocity 4-vector τ^μ: $\tau^\mu = dx^\mu/ds$, $\tau^\mu \tau_\mu = 1$. This vector field can be used to obtain time-like and space-like observables in a tangent space. Thus, the time interval in this frame of reference is defined as

$$dt = \tau_\mu dx^\mu \tag{3}$$

If $g_{\mu\nu}$ is the metric tensor, then

$$b_{\mu\nu} = \tau_\mu \tau_\nu - g_{\mu\nu} \tag{4}$$

is orthogonal to τ^μ: $\tau^\mu\, b_{\mu\nu} = 0$ and can be used as an operator of projecting four-dimensional objects onto a three-dimensional space-like hypersurface orthogonal to time. The metric can now be defined in terms of physical time and space intervals as

$$ds^2 = dt^2 - dl^2 \tag{5}$$

where $dl^2 = b_{\mu\nu}dx^{\mu}dx^{\nu}$. According to this so-called τ-field approach [17], [18], [19], [20], [21], [22], any 4-vector A_{μ} can be decomposed into observable time and space components: $a=A_{\mu}\tau^{\mu}$ and $a_{\mu}=A^{\nu}b_{\mu\nu}$.

These physically observable quantities can only be obtained in a frame of reference and are only meaningful in this frame, which further underscores the epistemological significance of the frame of reference.

2. What is a Frame of Reference?

A. Evolution of the Frame of Reference Concept

Let us briefly review the evolution of the concept of the frame of reference. In Newtonian mechanics, reference frames play a very important role although the concept is not rigorously defined. It is implicitly understood that a frame of reference is comprised of a reference body, which is taken to be a rigid body, with a Cartesian coordinate system and a clock rigidly attached to the reference body. Galilean transformations supply the laws of transformation between IFRs. The IFR is defined as a FR whose reference body is the state of rest or inertial movement, i.e. free from any forces. This definition, of course, is an idealization as one can never be sure that the reference body is free from any forces. Einstein questioned whether there exists an inertial frame of reference. [23],[24]. This concept can rather be defined as an approximation wherein the measured deviation from the second law of Newton is less than the measurement error, i.e., it is undetectable ([23], p.58). Einstein considered General Relativity as a theory that did away with the notion of the IFR, which he considered one of the greatest accomplishments of this theory.

The NIFR are dealt with in Newtonian mechanics by adding an *ad hoc* term to the second law of mechanics, which describes the inertial forces, such as centrifugal or Carioles forces.

In Special Relativity, the concept of the frame of reference undergoes a substantial revision by way of combining space and time into a unified spacetime continuum called Minkowski space (which is a pseudo-Euclidean space) with the resulting replacement of the Galilean transformation by the Lorentz transformation. The Theory of Special Relativity is a relativistic theory of IFR and it does not explicitly address the NIFR. Attempts, however, have been made to utilize Special Relativity for description of specialized NIFR (see, for example, [29], pp 74-77; [1], pp 163-176; and [25], pp 9-13). For a FR with uniform acceleration, this description leads to a hyperbolic movement in Rindler space severely limited by the event horizon. To quote MTW, “It is very easy to put together the words ‘the coordinate system of accelerated observer’…if taken seriously, they are self-contradictory” ([1], p.168).

B. Coordinate Systems

It was in search of the description of NIFR that Einstein turned to curvilinear coordinate systems replacing the Lorentz transformation with a general coordinate transformation. The requirement of general covariance, which should be a minimal requirement for any mathematical model purporting to describe some aspect of physical reality, was elevated to the exalted status of Principle of General Relativity.

As Rodichev pointed out ([12], p.287), even in Newtonian mechanics the transformation from one frame of reference to another is described by the fully covariant equation

$$r(t) = a(t) + r'(t), \qquad t = t' \tag{6}$$

where 3-vectors ***r*** and ***r'*** denote the position of a test particle in IFR and NIFR, respectively. Obviously, this expression does not depend on the choice of the coordinate system.

It may be one of those curious cases in the history of science when a wrong idea led to one of the most beautiful theories – the General Theory of Relativity. Kretschman was the first to realize that the principle of general covariance had nothing to do with general relativity and has no physical and very little geometrical meaning [2]. Einstein seemed to agree with this criticism but noted that, "Even though it is true that one must be able to bring every empirical law into general covariant form, yet the Principle [of general covariance – AP] has considerable heuristic force, which proved itself in the problem of Gravitation." (see [15], p.373). The only physical (or, rather, topological) meaning coordinates have is that they reflect the dimensionality of physical space ([22], p.67).

The transformation from one reference frame to another is accomplished by a general coordinate transformation on V_4, which Rodichev calls transformation of group A having physical meaning of the calibration of the measurement scale [9], [10], [11], [12].

Attempts have been made to separate those coordinate transformations that are time dependent. Such transformations are thought to be suitable to the description of the transformation from one FR to another. In truth, it is nothing more than a time-dependent scheme to number the points of the manifold, which, of course, has nothing to do with the physical motion of the reference body. The fallacy of this approach, if it is not self-evident, has been convincingly demonstrated by Rodichev [9], [10], [11], [12].

Unfortunately, many elegant schemes to describe the frame of reference by making use of chronometric [19], [20] and kinemetric [26], [27] invariants at the end of the day fall into the same trap of mistaking a time-dependent numbering scheme for the real reference frame transformation.

A simple and convincing argument in this debate is that the inertial force arising in any NIFR is a true vector. Obviously, in an IFR the inertial force is zero. However, if a vector is equal to zero in one coordinate system, it is equal to zero in all coordinate systems, and if it is not zero in one coordinate system, it is not zero in all coordinate systems. Consequently there exists no coordinate transformation that can transform a zero vector in an IFR into a non-zero vector in a NIFR. This conclusively proves that coordinate transformation cannot describe a transition from an IFR to a NIFR.

It is interesting to note that Einstein understood early on that non-inertial frames of reference may involve non-Euclidean geometry. For example, his example of a rotating disk is well-known. The diameter of the disk does not experience any relativistic contraction because there is no radial component in the velocity of rotation. On the other hand, the circumference of the disk will experience a relativistic contraction due to the linear velocity. Thus, the ratio of the circumference to the diameter will be greater than π, which indicates the curvature of space (see, for example, [29], pp.222).

C. Monads or $\boldsymbol{\tau}$ –Field

The monad or so-called $\boldsymbol{\tau}$-field approach has been already described above when we discussed the problem of measurement. It aims to separate space and time from the spacetime continuum to obtain physically observable quantities. The foundation of this approach rests on the representation of the FR as a congruence of worldlines of various points of the reference body associated with the given FR. This congruence of the worldlines is invariant with respect to the general coordinate transformation (Group A) and can be represented by the vector field of 4-velocities tangent to these worldlines – the $\boldsymbol{\tau}$-field.

Unfortunately, further conditions applied to the τ- field, such as respectively so-called chronometric [19] and kinematic [27], [28] conditions:

$$\tau^{\mu} = \frac{g_0^{\mu}}{\sqrt{g_{00}}}, \tau_{\mu} = \frac{g_{\mu}^{0}}{\sqrt{g^{00}}} \qquad (7)$$

spoil the invariant nature of the τ- field.

An even more serious difficulty of this approach is that it describes the FR by the field of 4-velocities of its reference body measured in another (presumably inertial) FR. Since the observer in a given FR cannot measure his own $\boldsymbol{\tau}$-field, as he is at rest with respect to the reference body, i.e., the $\boldsymbol{\tau}$-field is identically zero for this observer everywhere, it is of little use to the observer. As we see here, authors often implicitly presuppose the existence of some Minkowski space, in which the worldlines of the moving reference bodies are drawn, or they use an IFR in which they define a frame of reference. If Minkowski space is not associated with some IFR, it becomes an absolute space in contradiction to the relativity principle. On the other hand, using one reference frame to define another reference frame, such as using the velocity of a NIFR calculated with respect to an IFR to define the NIFR, is a circular logic. This circular definition is characteristic of most approaches to reference frames.

D. Tetrads

In this approach the frame of reference is identified with the set of four vectors $\mathbf{e}_{(a)}=\{\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ defined in any given point of the differentiable manifold, called a tetrad. The tetrad, which is a special case of Cartan's *repère mobile*, is a basis, which may be chosen to coincide with a coordinate basis, i.e. an infinitesimal coordinate system defined by the four linearly-independent vectors $\mathbf{e}_{(i)}{}^{\mu}$, where (i) is the number of the vector and μ is the regular tensor coefficient denoting a particular component of this contravariant vector in a local chart. Usually the tetrad is comprised of the basis vectors orthogonal to each other, in which case the tetrad is called orthonormal. The 0^{th} vector of the tetrad, $\mathbf{e}_{(0)}{}^{\mu}$, is usually selected to be tangent to the worldline of the observer, in which case it is the timelike 4-velocity vector of the reference body.

$$e^{\mu}_{(0)} = \tau^{\mu} \equiv \frac{dx^{\mu}}{ds} \tag{8}$$

The tetrad is invariant with respect to the general coordinate transformation (Group A). The transition from one reference frame to another is described in this approach by the tetrad transformation (Group B):

$$e_{(i')} = \omega^{(k)}_{(i')} e_{(k)} \tag{9}$$

There are different interpretations of the physical meaning of the tetrad. According to Rodichev ([12], p.300), a tetrad defines the three Euler angles and the velocity of the center mass, six values in total. According to others (see for example [22], p.72), the additional degrees of freedom describe the spin of the particles forming the reference body.

This tetrad moves along the worldline of the observer by means of the Fermi-Walker transport. This preserves the orthogonal orientation of the timelike vector of 4-velocity to the other three spacelike vectors of the tetrad.

The tetrad transformation (Group B) does not change any of the physical vectors of the basis, which may be implemented by gyroscopes. It only changes the initial values for the velocity of the center-mass and three Euler angles. As these values could be selected arbitrarily, according to Rodichev, Group B transformation has as much physical meaning as Group A coordinate transformation and, consequently, cannot describe physical transition from one FR to another. Thus the laws of physics must be invariant with respect to Group B transformation as they must be invariant with respect to Group A coordinate transformation.

Instead of coordinate tetrads, Rodichev proposed to use the invariant tetrads and instead of Group B transformation, to use the affinor transformation representing transition from one FR to another:

$$
\begin{aligned}
u_{(a)} &= \Omega_{(a)(b)} u'_{(b)} \\
u'_{(a)} &= \tilde{\Omega}_{(a)(b)} u_{(b)}
\end{aligned}
\tag{10}
$$

As one would expect, Rodichev arrives at a curved spacetime in a NIFR.

The invariant tetrad field approach, albeit the most comprehensive, appears to be overkill. It is hardly reasonable to suppose that a reference body of a FR consists of a plurality of particles, each having its own velocity, acceleration, spatial orientation, spin, etc. In our view, only one such particle, or a rigid body made of many particles which all move in unison, can be considered as a reference body and define the FR. The motion of all other particles would have to be considered as moving with respect to this FR.

E. Definition

A physical definition of a FR has to include the following elements:

- An observer
- A reference body
- A standard of length
- Gyroscopes
- A clock
- An accelerometer

An observer, who plays a very important role in quantum physics, is usually omitted from consideration in Newtonian or relativistic mechanics. Yet the observer, as we will show, plays a crucial role in classical physics as well – without a coordinative definition that is a free choice of a conscious observer the geometry of nature is indeterminable, as Reichenbach has demonstrated [16].

As we pointed out before, it is not reasonable to allow a reference body to be a plurality of independent particles each having its own velocity, acceleration, spatial orientation, etc. We require that a reference body be a single rigid body, such as a person's own body (which, of course, is not rigid, but for the purposes of this discussion may be approximated as a point-mass), laboratory or a spacecraft. All other objects not rigidly connected to the reference body will be considered moving with respect to this FR.

An observer in any FR will need a rigid rod as a standard of length, such as meter; he will require gyroscopes to set physical directions in space and to detect a possible rotation of the reference body rendering this FR noninertial; he will require a clock to measure his proper time and an accelerometer to detect possible acceleration that would also render the FR noninertial.

A necessary condition is that all of these elements be rigidly connected to the reference body, i.e. be immobile with respect to the walls of the laboratory or the spacecraft.

F. Mathematical Representation

It is clear that the observer in a given FR defines his or her own manifold. We will assume that it is differentiable manifold – a four-dimensional spacetime continuum. With each FR defining its own manifold, the question arises as to the correlation between such manifolds. A faith in the objective existence of the physical world would lead us to believe that these manifolds are homeomorphic. However, we must remember that an event perceivable in one FR may not necessarily be perceivable in another if it lies outside its event horizon. Rindler space is a well-known example of such a possibility ([25], p.11). Having noted that, for simplicity, we will consider the manifold generated by the chosen FR to be the common differentiable manifold for all FRs subject to the limitations of their particular event horizons. This simply means that all observers observe the same events albeit from their peculiar vantage point – FR.

Each individual FR will generate its own geometry on this differentiable manifold. The question remains: how does a particular FR generate a geometric structure on the differentiable manifold? Referring to our physical definition of the FR given above, we see that the length standard which allows the observer to measure the distance between two points sets up the metric $\boldsymbol{g}$ on the manifold. To define parallel transport of vectors, which is essential, an affine connection Γ is required. Fortunately, it is naturally (although not uniquely) generated in any FR. As we observe the trajectories of free-moving test particles and require these trajectories to be geodesic lines of the spacetime defined by the chosen FR, such geodesic lines define the affine connection up to a geodesic transformation and torsion. Indeed, a freely moving test particle in an IFR moves along a geodesic line of Minkowski space:

$$\frac{d^2x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0 \tag{11}$$

where Γ is a flat Levi-Civita connection of Minkowski space and τ is an affine parameter along this line.

In this IFR a reference body accelerating with acceleration $\boldsymbol{a}$ moves along a worldline

$$\frac{d^2x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = a^\lambda \tag{12}$$

that is no longer geodesic.

In the accelerating NIFR a test particle moves in the opposite direction with acceleration $-\boldsymbol{a}$:

$$\frac{d^2x^\lambda}{dt^2}+\Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}=-a^\lambda \tag{13}$$

Determined to eliminate the universal forces, the observer in the NIFR demands that this trajectory represent the geodesic line of a non-Euclidean space:

$$\frac{d^2x^\lambda}{dt^2}+\overline{\Gamma}^\lambda_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}=0 \tag{14}$$

where $\overline{\Gamma}$ is an affine connection on the differential manifold and t is an affine parameter along this line.

Since the two expressions (13) and (14) represent the same trajectory of the test particle, we can subtract (13) from (14) to obtain

$$T^\lambda_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}=a^\lambda \tag{15}$$

where

$$T^\lambda_{\mu\nu}=\overline{\Gamma}^\lambda_{\mu\nu}-\Gamma^\lambda_{\mu\nu} \tag{16}$$

is called the affine deformation tensor. If $\eta_{\mu\nu}$ is the Minkowski metric tensor, we can choose

$$T^\lambda_{\mu\nu}=a^\lambda\eta_{\mu\nu} \tag{17}$$

At this point there is no motivation to impose a Riemannian requirement that the covariant derivative of the metric vanishes identically and, therefore, the metric and the connection remain totally independent. Hence we have the geometric structure of the space of affine connection with an independent metric, sometimes called Weyl space W_n or (L_n,g)-space. The affine connection has the form:

$$\overline{\Gamma}^\lambda_{\mu\nu}=\Gamma^\lambda_{\nu\mu}+\mathrm{T}^\lambda_{\nu\mu} \tag{18}$$

It is easy to see that Minkowski metric $\boldsymbol{\eta}$ is inhomogeneous with respect to the affine connection $\overline{\Gamma}$, i.e. its covariant derivative with respect to this connection does not vanish: $h_{\mu\nu;\lambda}\neq 0$, where the semicolon denotes a covariant derivative with respect to the affine connection $\overline{\Gamma}$. Generally, the affine deformation $T^\lambda{}_{\mu\nu}$ is comprised of the symmetric tensor of nonmetricity $Q^\lambda{}_{\mu\nu}$ and anti-symmetric torsion tensor $S^\lambda{}_{\mu\nu}$:

$$T^\lambda_{\mu\nu}=S^\lambda_{\mu\nu}+Q^\lambda_{\mu\nu} \tag{19}$$

As is known, torsion does not affect geodesics, i.e. two affine connections different only by torsion have the same geodesics. Thus, for forward acceleration not involving rotation of the reference body about its axis, we can disregard torsion and assume that affine connection $\bar{\Gamma}^{\lambda}_{\mu\nu}$ is symmetric in its two lower indexes:

$$\bar{\Gamma}^{\lambda}_{\mu\nu} = \bar{\Gamma}^{\lambda}_{\nu\mu} \tag{19}$$

and, therefore, the tensor of affine deformation is equal to the tensor of nonmetricity:

$$T^{\lambda}_{\mu\nu} = Q^{\lambda}_{\mu\nu} \tag{20}$$

The tensor of nonmetricity can be expressed through the covariant derivatives of the metric tensor $\boldsymbol{\eta}$ as follows:

$$\eta_{\tau\sigma} Q^{\tau}_{\mu\nu} = \frac{1}{2}\left(\nabla_{\mu}\eta_{\nu\sigma} + \nabla_{\nu}\eta_{\mu\sigma} - \nabla_{\sigma}\eta_{\mu\nu}\right) \tag{21}$$

It is important to note that our approach to definition of NIFR does not suffer from the circularity of argument as many other approaches, such as coordinate systems or monads, do. Indeed, the equations (11) and (12) were only used for illustration purposes and are not essential to the argument. Since an observer in a NIFR can measure the acceleration of test particles in his reference frame directly (and can measure the acceleration of his reference body by use of accelerometers and gyroscopes) we may start directly from the equation (13) where the acceleration $\boldsymbol{a}$ is measured within the NIFR and the choice of connection (flat Levi-Civita connection Γ of Minkowski space or affine connection $\bar{\Gamma}$ of (L_n,g) space) is a matter of coordinative definition. Each can be chosen within the NIFR and we do not need a background spacetime or an IFR to define a NIFR.

We can derive from this analysis the following conclusion:

a. The spacetime in a frame of reference is generally a (L_n,g) metric-affine space with independent affine connection $\bar{\Gamma}$ and inhomogeneous metric $\boldsymbol{\eta}$.
b. In an IFR where nonmetricity vanishes, the affine connection is a trivial (flat) Levi-Civita connection compatible with the Minkowski metric $\boldsymbol{\eta}$.
c. In a NIFR the affine connection $\bar{\Gamma}$ has curvature (and may have torsion); however, this curvature is due to the nonmetricity of this connection.
d. Any two FRs share the same metric but have different affine connections.
e. A transformation from an IFR to a NIFR, or from one NIFR to another, amounts to affine deformation of the connection.

3. General Relativity in a Spacetime with Affine Connection and Metric

In 1980, we proposed that the frame of reference is described by a differentiable manifold with an affine connection $\overline{\Gamma}$ and metric $\boldsymbol{g}$ [13]. We showed that a Levi-Civita connection Γ can be decomposed into its affine $\overline{\Gamma}$ and nonmetric $\mathbf{Q}$ components: $\Gamma = \overline{\Gamma} + \mathbf{Q}$. This in turn leads to a unique decomposition of the Riemannian curvature and Einstein's curvature tensors into the sum of affine and nonmetric components:

$$\begin{aligned} \mathbf{R} &= \overline{\mathbf{R}} + \tilde{\mathbf{R}} \\ \mathbf{G} &= \overline{\mathbf{G}} + \tilde{\mathbf{G}} \end{aligned} \tag{22}$$

where $\mathbf{R}$ is the Riemannian curvature tensor, $\overline{\mathbf{R}}$ is its affine component and $\tilde{\mathbf{R}}$ is its nonmetric component; and $\mathbf{G}$ is the Einstein tensor:

$$G_{\mu\nu} \equiv R_{\mu\nu} + \frac{1}{2} R g_{\mu\nu} \tag{23}$$

$\overline{\mathbf{G}}$ is its affine component:

$$\overline{G}_{\mu\nu} \equiv \overline{R}_{\mu\nu} + \frac{1}{2} \overline{R} g_{\mu\nu} \tag{24}$$

and $\tilde{\mathbf{G}}$ is its nonmetric component:

$$\tilde{G}_{\mu\nu} \equiv \tilde{R}_{\mu\nu} + \frac{1}{2} \tilde{R} g_{\mu\nu} \tag{25}$$

This decomposition allowed us to recast the Einstein gravitational field equations in a form invariant with respect to the arbitrary choice of a FR:

$$\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{R} g_{\mu\nu} + \frac{1}{2} \overline{R} g_{\mu\nu} = 8\pi T_{\mu\nu} - \overline{R}_{\mu\nu} \tag{26}$$

We see that in sharp contrast with General Relativity, where gravity is described by the metric, in this equation gravity is represented by nonmetricity (tensor of nonmetricity $\mathbf{Q}$ plays the role of the strength of the gravitational field). It is important to stress that this is not an alternative theory of gravitation. The field equation (26) was obtained from the standard Einstein field equation. We merely separated the contribution of the inertial forces from the gravity by fixing the affine connection representing a chosen frame of reference.

Equation (26) can be recast in the following form:

$$\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{R} g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi \tilde{T}_{\mu\nu} \tag{27}$$

where

$$\Lambda = \frac{1}{2}\bar{R} \tag{28}$$

and

$$\tilde{T}_{\mu\nu} = T_{\mu\nu} - \frac{1}{8\pi}\bar{R}_{\mu\nu} \tag{29}$$

We can make two interesting observations about equation (27): (a) the scalar curvature of the affine connection plays the role of the Einstein cosmological constant (which in our case does not have to be constant and is more akin to a dynamic field like quintessence) and (b) the Ricci tensor of the affine connection contributes to the energy-momentum tensor as an additional field source. This is not at all unexpected. The inertial forces existing in a NIFR have energy and therefore must contribute to the gravitational field. What is noteworthy is that these inertial forces are *repulsive* forces (akin to the centrifugal force) and counteract the attractive pull of the gravitational forces. Therefore, the inertial forces accelerate the expansion of the universe, which is exactly what is presently observed. The accelerated expansion of the universe implies that we are observing the universe from a NIFR. This may be an indication that the mysterious repulsive dark energy pervading the universe is but a field of inertial forces arising out of the noninertial frame of reference in which we see our world. Whether or not this explanation proves satisfactory, its very possibility underscores the extraordinary role frames of reference play in the ontology of spacetime.

IV. Conclusion

As we have shown, various existing approaches to the description of frames of reference are unacceptable as they rely on circular logic or preexistent absolute space. A new mathematical model of a reference frame was proposed based on a differential manifold with an independent affine connection and metric, i.e., so called metric-affine (L_4,g) space. The affine connection is determined by observation of the trajectories of free-moving test particles. It is the role of the observer in any given frame of reference to choose coordinative definitions that eliminate such universal forces as gravity and noninertial forces thereby opting for a non-Euclidean geometry. The transformation between various reference frames is described as affine deformation.

Fixing a reference frame of an observer leads to a novel view of the spacetime in GR as affine connection geometry with independent metric wherein the gravity is described as nonmetricity of the spacetime. Incidentally, this approach leads to a fully covariant theory of gravitation and the solution of the Energy Problem in GR [14]. It is suggested that this approach may prove fruitful in explaining the nature of the cosmological constant and dark energy in terms of inertial forces.

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